International Journal of Theoretical Physics, Vol. 33, No. 6, 1994

Transformation Properties of a Constrained Hamiltonian System and PBRST Charge

Zi-ping Li[1]

Received April 10, 1993

We derive a generalized first Noether theorem for weakly quasi-invariant systems with singular higher-order Lagrangians, subject to the extra constraints and generalized Noether identities for a variant system in phase space. The strong and weak conservation laws for variant systems are also deduced. Some preliminary applications to field theories are given. In certain cases a variant system is also a constrained Hamiltonian system. A PBRST (weak) conserved charge is obtained that differs from the usual BRST charge.

Symmetry is one of the most significant concepts in modern theoretical physics. Noether theorems refer to the invariance of a system. In previous papers we gave a generalized first Noether theorem (GFNI) for constrained and nonconservative systems, and generalized Noether identities (GNI) for variant systems (Li, 1981, 1984, 1985, 1987, 1988; Li and Li, 1990). In these papers the Lagrangian is expressed in configuration space, and corresponding transformations are given in terms of Lagrange's variables. For systems with regular Lagrangians in classical mechanics, the invariance under a finite continuous group in terms of Hamilton's variables was discussed by Djukic (1974). A presymplectic version of Noether's theorem for a constrained system was proved (Ferrario and Passerini, 1990). The extended second Noether theorem was discussed by Lusanna (1991) for weakly quasi-invariant systems. A system with singular Lagrangian is subject to some inherent phase space constraints (Dirac, 1964). The generalization of Noether theorems in canonical variables for systems of finite degrees of freedom with singular Lagrangian was given by Li and Li (1991), and

[1]CCAST (World Laboratory), P.O. Box 8730, Beijing, 100080, China, and Department of Applied Physics, Beijing Polytechnic University, Beijing, 100022, China.

systems with singular higher-order Lagrangian were also investigated (Li, 1991). Here the transformation properties of singular Lagrangians in terms of canonical variables for field theories (including higher-order derivatives) are further investigated. Theories with higher-order derivatives (Galvão and Pimentel, 1988; Saito *et al.*, 1989) exhibit many interesting aspects.

Consider a system described by a Lagrangian function depending on a set of field variables $\psi(x) \equiv \{\psi^\alpha(x)\}$, $\alpha = 1, 2, \ldots, N$, and their first and second derivatives with respect to the space-time coordinates, $\partial\psi(x) \equiv \psi_{,\mu} \equiv \{\partial_\mu \psi^\alpha(x)\}$, $\partial^2\psi \equiv \psi_{,\mu\nu} \equiv \{\partial_\mu\partial_\nu\psi^\alpha(x)\}$, $\mu, \nu = 0, 1, 2, 3$. The flat space-time metric is $\eta_{\mu\nu} = \mathrm{diag}(+ \ - \ - \ -)$. Let $\mathscr{L} = \mathscr{L}(\psi, \partial\psi, \partial^2\psi)$ be the Lagrangian density, which is called singular (or degenerate) if the Hessian matrix is degenerate

$$\det|H_{\alpha\beta}| = \det\left|\frac{\partial^2 \mathscr{L}}{\partial\ddot{\psi}^\alpha \partial\ddot{\psi}^\beta}\right| = 0 \tag{1}$$

The Ostrogradsky transformation introduces canonical momenta (Galvão and Pimentel, 1988)

$$\pi_\alpha^{(0)} = \frac{\partial\mathscr{L}}{\partial\dot{\psi}^\alpha} - 2\partial_k\left(\frac{\partial\mathscr{L}}{\partial\psi^\alpha_{,k0}}\right) - \partial_0\left(\frac{\partial\mathscr{L}}{\partial\ddot{\psi}^\alpha}\right) \tag{2a}$$

$$\pi_\alpha^{(1)} = \frac{\partial\mathscr{L}}{\partial\ddot{\psi}^\alpha} \tag{2b}$$

One can then go over from the Lagrangian description to the Hamiltonian description. The canonical Hamiltonian density is given by

$$\mathscr{H}_c = \pi_\alpha^{(0)}\dot{\psi}^\alpha_{(0)} + \pi_\alpha^{(1)}\dot{\psi}^\alpha_{(1)} - \mathscr{L} \tag{3}$$

where $\psi^\alpha_{(0)} = \psi^\alpha$, $\psi^\alpha_{(1)} = \dot{\psi}^\alpha$. Suppose the rank of the Hessian matrix is $N - R$, one cannot solve for all $\ddot{\psi}^\alpha$ from (2b), the definition of canonical momenta, because of (1). This implies the existence of constraints (Galvão and Pimentel, 1988)

$$\phi_a^0(\psi^\alpha_{(r)}, \pi_\alpha^{(r)}) \approx 0, \qquad r = 0, 1, \quad a = 1, 2, \ldots, R < N \tag{4}$$

which are called primary constraints (PC), and $\approx$ is the symbol for weak equality, which indicates that one is allowed to impose the constraints only after calculating any Poisson bracket.

The canonical action for the system is given by

$$I = \int_\Omega \mathscr{L}_p \, d^4x = \int_\Omega (\pi_\alpha^{(r)}\dot{\psi}^\alpha_{(r)} - \mathscr{H}_c) + d^4x \tag{5}$$

Let us consider the transformation properties of the system under the continuous group with the infinitesimal transformation given by

$$x^{\mu'} = x^\mu + \delta x^\mu$$
$$\psi^\alpha_{(r)}(x') = \psi^\alpha_{(r)}(x) + \delta\psi^\alpha_{(r)}(x) \tag{6}$$
$$\pi^{(r)}_\alpha(x') = \pi^{(r)}_\alpha(x) + \delta\pi^{(r)}_\alpha(x)$$

In (6), $\delta\psi^\alpha_{(r)}$, $\delta\pi^{(r)}_\alpha$ are total variations; they can be expressed in terms of the local variations $\bar{\delta}\psi^\alpha_{(r)}, \bar{\delta}\pi^{(r)}_\alpha$. $\delta\psi^\alpha_{(r)} = \bar{\delta}\psi^\alpha_{(r)} + \psi^\alpha_{,\mu}\delta x^\mu$ and $\delta\pi^{(r)}_\alpha = \bar{\delta}\pi^{(r)}_\alpha + \pi^{(r)}_{\alpha,\mu}\delta x^\mu$. Suppose the variation of the Lagrangian $\mathscr{L}_p$ is given by $\delta\mathscr{L}_p = \partial_\mu\Lambda^\mu + W$ under the transformation (6); we have

$$\int_\Omega \left[\frac{\delta I}{\delta\pi^{(r)}_\alpha}\bar{\delta}\pi^{(r)}_\alpha + \frac{\delta I}{\delta\psi^\alpha_{(r)}}\bar{\delta}\psi^\alpha_{(r)}\right] d^4x$$
$$= \int_\Omega [\partial_\mu(\Lambda^\mu - \mathscr{L}_p\delta x^\mu) - \frac{d}{dt}(\pi^{(r)}_\alpha\bar{\delta}\psi^\alpha_{(r)}) + W]\, d^4x \tag{7}$$

where

$$\frac{\delta I}{\delta\pi^{(r)}_\alpha} = \dot{\psi}^\alpha_{(r)} - \frac{\delta H_c}{\delta\pi^{(r)}_\alpha}, \qquad \frac{\delta I}{\delta\psi^\alpha_{(r)}} = -\dot{\pi}^{(r)}_\alpha - \frac{\delta H_c}{\delta\psi^\alpha_{(r)}} \tag{8}$$

and $H_c = \int_\Omega \mathscr{H}_c\, d^4x$.

For a finite continuous group G_s, let $\delta x^\mu = \varepsilon_\sigma\tau^{\mu\sigma}(x, \psi^\alpha_{(r)}, \pi^{(r)}_\alpha)$, $\delta\psi^\alpha_{(r)} = \varepsilon_\sigma\xi^{\alpha\sigma}_{(r)}(x, \psi^\alpha_{(r)}, \pi^{(r)}_\alpha)$, $\delta\pi^{(r)}_\alpha = \varepsilon_\sigma\eta^{(r)\sigma}_\alpha(x, \psi^\alpha_{(r)}, \pi^{(r)}_\alpha)$, and $\Lambda^\mu = \varepsilon_\sigma\Lambda^{\mu\sigma}(x, \psi^\alpha_{(r)}, \pi^{(r)}_\alpha)$, where ε_σ ($\sigma = 1, 2, \ldots, s$) are parameters. Suppose that for a weakly quasi-invariant system under this transformation (6), $W \mathrel{\mathring{=}} 0$ ($\mathring{=}$ means "evaluated on the trajectory of motion") and the variation of the constraint conditions (4) are given by

$$\bar{\delta}\phi^0_a = \frac{\partial\phi^0_a}{\partial\psi^\alpha_{(r)}}\bar{\delta}\psi^\alpha_{(r)} + \frac{\partial\phi^0_a}{\partial\pi^{(r)}_\alpha}\bar{\delta}\pi^{(r)}_\alpha \approx 0 \tag{9}$$

These conditions imply that the constraints are invariant under the local variation induced by (6). Introducing a set of Lagrange multipliers $\lambda^a(x)$, combining the expressions (7) and (9), and using the equations of motion of a constrained Hamiltonian system (Gitman and Tyutin, 1990)

$$\dot{\psi}^\alpha_{(r)} = \frac{\delta H_T}{\delta\pi^{(r)}_\alpha}, \qquad \dot{\pi}^{(r)}_\alpha = -\frac{\delta H_T}{\delta\psi^\alpha_{(r)}} \tag{10}$$

with

$$H_T = H_c + H' = \int (\mathscr{H}_c + \lambda^a\phi^0_a)\, d^3x \tag{11}$$

one obtains the following GFNT in the canonical formalism: If under the transformation (6) the phase-space Lagrangian $\mathscr{L}_p = \pi_\alpha^{(r)}\dot{\psi}_{(r)}^\alpha - \mathscr{H}_c$ is weakly quasi-invariant (i.e., $\delta\mathscr{L}_p \overset{\circ}{=} \partial_\mu \Lambda^\mu$) and the constraints are invariant under the local variation determined by (6), then there are s constants of motion

$$\int_V (\pi_\alpha^{(r)} \xi_{(r)}^{\alpha\sigma} - \mathscr{H}_c \tau^{0\sigma} - \Lambda^{0\sigma})\, d^3x = \text{const} \qquad (\sigma = 1, 2, \ldots, s) \tag{12}$$

This result is a generalization of regular and singular Lagrangian systems with finite degrees of freedom (Li and Li, 1991; Li, 1991).

From the stationarity of the PC, one can define successively the secondary constraints (SC) according to the Dirac–Bergmann algorithm. All the constraints are classified into first class and second class. Dirac conjectured that all secondary first-class constraints (SFCC) are independent generators of the gauge transformation which generates equivalence transformations among physical states. If this conjecture holds true, then the dynamics of a constrained Hamiltonian system should be correctly described by the equations of motion arising from the extended Hamiltonian $H_E = H_T + \mu^a \chi_a$, where χ_a are SFCC and μ^a are Lagrange multipliers. There have been some objections to Dirac's conjecture, and some counterexamples have been given. All these objections are based on the straightforward observation that the equations of motion derived from the extended Hamiltonian H_E are not strictly equivalent to the corresponding Lagrange equations. Based on the symmetry properties in phase space for the constrained Hamiltonian system, we can consider whether the conservation laws derived from H_E via the canonical formalism are equivalent to the results arising from Lagrange's formalism via the classical Noether theorem. We presented an example for a system with finite degrees of freedom in which Dirac's conjecture fails (Li, 1991) in which we did not write the constraints in linearized form as Cawley and others do. This implies that the dynamics of a constrained Hamiltonian system should be described by the equations of motion deriving from the total Hamiltonian H_T.

Similar discussion can be given for field theories.

The GFNT in the canonical formalism can be easily extended to the case when the system is subjected to the extra constraint $f_b(x, \psi^\alpha) = 0$ ($b = 1, 2, \ldots, M, R + M < N$) and the equations of motion are derived from $H_T' = H_T + \int \mu^b f_b\, d^3x$, where $\mu^b(x)$ are also Lagrange multipliers. In this case one need further to require that the extra constraints are invariant under the local variation determined by (6).

Now consider the infinite continuous group $G_{\infty s}$, and let $\delta x^\mu = R_\sigma^\mu \varepsilon^\sigma(x)$, $\delta\psi_{(r)}^\alpha(x) = S_{(r)\sigma}^\alpha \varepsilon^\sigma(x)$, $\delta\pi_\alpha^{(r)}(x) = T_{\alpha\sigma}^{(r)} \varepsilon^\sigma(x)$, $\Lambda^\mu = \Lambda_\sigma^\mu \varepsilon^\sigma(x)$, and $W =$

$U_\sigma \varepsilon^\sigma(x)$, where $\varepsilon^\sigma(x)$ are arbitrary functions $(\sigma = 1, 2, \ldots, s)$, and R^μ_σ, $S^\alpha_{(r)\sigma}$, $T^{(r)}_{\alpha\sigma}$, Λ^μ_σ, and U_σ are linear differential operators:

$$\begin{gathered} R^\mu_\sigma = a^{\mu\nu(j)}_\sigma \partial_{\nu(j)}, \qquad S^\alpha_{(r)\sigma} = b^{\alpha\nu(k)}_{(r)\sigma} \partial_{\nu(k)}, \qquad T^{(r)}_{\alpha\sigma} = C^{(r)\nu(l)}_{\alpha\sigma} \partial_{\nu(l)} \\ \Lambda^\mu_\sigma = e^{\mu\nu(m)}_\sigma \partial_{\nu(m)}, \qquad U_\sigma = u^{\mu(n)}_\sigma \partial_{\mu(n)} \end{gathered} \tag{13}$$

where

$$U^{\mu(n)}_\sigma = U^{\overbrace{\mu\nu\lambda\cdots\sigma\rho}^{n}}, \qquad \partial_{\mu(n)} = \underbrace{\partial_\mu \partial_\nu \partial_\lambda \cdots \partial_\sigma \partial_\rho}_{n} \tag{14}$$

and a, b, c, e, and u are functions of x, $\psi^\alpha_{(r)}$, and $\pi^{(r)}_\alpha$. From the identity (7), one can choose $\varepsilon^\sigma(x)$ and their derivatives up to a required order to vanish on the boundary of a domain, and then we can make the boundary terms of the right-hand side of the identity (7) vanish. We repeat the integral by parts of the remaining terms of this identity, after which we apply the fundamental lemma of the calculus of variation to conclude that the GNI in canonical variables can be written as

$$\begin{aligned} &\tilde{T}^{(r)}_{\alpha\sigma}\left(\frac{\delta I}{\delta \pi^{(r)}_\alpha}\right) - \tilde{R}^\mu_\sigma\left(\pi^{(r)}_{\alpha,\mu}\frac{\delta I}{\delta \pi^{(r)}_\alpha}\right) + \tilde{S}^\alpha_{(r)\sigma}\left(\frac{\delta I}{\delta \psi^\alpha_{(r)}}\right) - \tilde{R}^\mu_\sigma\left(\psi^\alpha_{(r),\mu}\frac{\delta I}{\psi^\alpha_{(r)}}\right) \\ &\quad = \tilde{U}_\sigma(1) \qquad (\sigma = 1, 2, \ldots, s) \end{aligned} \tag{15}$$

where $\tilde{R}^\mu_\sigma$, $\tilde{S}^\alpha_{(r)\sigma}$, $\tilde{T}^{(r)}_{\alpha\sigma}$, and $\tilde{U}_\sigma$ are adjoint operators with respect to R^μ_σ, $S^\alpha_{(r)\sigma}$, $T^{(r)}_{\alpha\sigma}$, and U_σ, respectively (Li, 1987, 1988).

As is well known, a gauge-invariant system in the Lagrangian formalism has a Dirac constraint. Using the GNI (15), we can further show that for certain variant systems there is also a Dirac constraint. Suppose in the expressions (13) we use

$$\begin{gathered} R^\mu_\sigma = a^\mu_\sigma, \qquad S^\alpha_{(r)\sigma} = b^\alpha_{(r)\sigma} + b^{\alpha\mu}_{(r)\sigma}\partial_\mu + b^{\alpha\mu\nu}_{(r)\sigma}\partial_\mu\partial_\nu \\ T^{(r)}_{\alpha\sigma} = c^{(r)}_{\alpha\sigma} + c^{(r)\mu}_{\alpha\sigma}\partial_\mu, \qquad U_\sigma = u_\sigma + u^\mu_\sigma\partial_\mu + u^{\mu\nu}_\sigma\partial_\mu\partial_\nu \end{gathered} \tag{16}$$

where a, b, c, and u are functions of x, $\psi^\alpha_{(r)}$, and $\pi^{(r)}_\alpha$, and $u^{\mu\nu}_\sigma$ are functions of x, ψ^α. The massive Yang–Mills theories with second-order Lagrangian belong to this category (Saito *et al.*, 1989). According to the GNI (15) and the definition of canonical momenta (2a), which leads to terms containing fifth-order time derivatives of ψ^α and must cancel each other irrespective of other terms (Li, 1991),

$$b^{\alpha 0}_{(r)\sigma} H_{\alpha\beta} \psi^\beta_{(5)} = 0 \tag{17}$$

These conditions are to be fulfilled for any fifth-order time derivatives of ψ^α; therefore one obtains

$$b^{\alpha 0}_{(r)\sigma} H_{\alpha\beta} = 0 \tag{18}$$

Since $b^{\alpha 0}_{(r)\sigma}$ are not all identically zero (for example, the gauge transformation), this implies $\det|H_{\alpha\beta}| = 0$; then the Hessian matrix is degenerate, and the system has a Dirac constraint.

Substituting the expression (16) into (7), one obtains

$$\left[\frac{\delta I}{\delta \pi^{(r)}_{\alpha}} (T^{(r)}_{\alpha\sigma} - \pi^{(r)}_{\alpha,\mu} a^{\mu}_{\sigma}) + \frac{\delta I}{\delta \psi^{\alpha}_{(r)}} (S^{\alpha}_{(r)\sigma} - \psi^{\alpha}_{(r),\mu} a^{\mu}_{\sigma})\right] \varepsilon^{\sigma}$$
$$= \partial_{\mu}[(\Lambda^{\mu}_{\sigma} - \mathscr{L}_p a^{\mu}_{\sigma})\varepsilon^{\sigma}] + (u_{\sigma} + u^{\mu}_{\sigma}\partial_{\mu} + u^{\mu\nu}_{\sigma}\partial_{\mu}\partial_{\nu})\varepsilon^{\sigma}$$
$$- \frac{d}{dt}[\pi^{(r)}_{\alpha}(S^{\alpha}_{(r)\sigma} - \psi^{\alpha}_{(r),\mu} a^{\mu}_{\sigma})\varepsilon^{\sigma}] \tag{19}$$

If $u^{\mu\nu}_{\sigma}$ are symmetric with respect to indexes μ and ν, multiplying (15) by ε^{σ} and subtracting the result from (19) yields

$$\partial_{\mu}\left[\left(b^{\alpha\mu}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}} + c^{(r)\mu}_{\alpha\sigma}\frac{\delta I}{\delta\pi^{(r)}_{\alpha}} + \partial_{\nu}\left(b^{\alpha\mu\nu}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}}\right) - \left(b^{\alpha\mu\nu}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}}\right)\partial_{\nu} + \mathscr{L}_p a^{\mu}_{\sigma} - \Lambda^{\mu}_{\sigma}\right.\right.$$
$$\left.\left. - u^{\mu}_{\sigma} - u^{\mu\nu}_{\sigma}\partial_{\nu} + \partial_{\nu} u^{\mu\nu}_{\sigma}\right)\varepsilon^{\sigma}\right] + \frac{d}{dt}[\pi^{(r)}_{\alpha}(S^{\alpha}_{(r)\sigma} - \psi^{\alpha}_{(r),\mu} a^{\mu}_{\sigma})\varepsilon^{\sigma}] = 0 \tag{20}$$

which leads to the strong conservation law

$$J = \int_V d^3x \left\{\left[b^{\alpha 0}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}} + c^{(r)0}_{\alpha\sigma}\frac{\delta I}{\delta\pi^{(r)}_{\alpha}} + \partial_{\nu}\left(b^{\alpha 0\nu}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}}\right)\right.\right.$$
$$- \left(b^{\alpha 0\nu}_{(r)\sigma}\frac{\delta I}{\delta\psi^{\alpha}_{(r)}}\right)\partial_{\nu} + \mathscr{L}_p a^{0}_{\sigma} - \Lambda^{0}_{\sigma} - u^{0}_{\sigma} - u^{0\nu}_{\sigma}\partial_{\nu} + \partial_{\nu} u^{0\nu}_{\sigma}$$
$$\left.\left. + \pi^{(r)}_{\alpha}(S^{\alpha}_{(r)\sigma} - \psi^{\alpha}_{(r),\mu} a^{\mu}_{\sigma})\right]\varepsilon^{\sigma}\right\} = \text{const} \tag{21}$$

Expression (21) is valid whether or not the equations of motion are satisfied. If the group $G_{\infty s}$ has a subgroup and $\varepsilon^{\sigma}(x) = \varepsilon^{\rho}_{0}\zeta^{\sigma}_{\rho}(x)$, where ε^{ρ}_{0} are numerical parameters of a continuous group, one gets the weak conservation law along the trajectory of motion,

$$J^{w}_{\rho} = \int_V d^3x \left\{\left[b^{\alpha 0}_{(r)\sigma}\frac{\delta H'}{\delta\psi^{\alpha}_{(r)}} + c^{(r)0}_{\alpha\sigma}\frac{\delta H'}{\delta\pi^{(r)}_{\alpha}} + \partial_{\nu}\left(b^{\alpha 0\nu}_{(r)\sigma}\frac{\delta H'}{\delta\psi^{\alpha}_{(r)}}\right) - \left(b^{\alpha 0\nu}_{(r)\sigma}\frac{\delta H'}{\delta\psi^{\alpha}_{(r)}}\right)\partial_{\nu}\right.\right.$$
$$\left.\left. + \mathscr{L}_p a^{0}_{\sigma} - \Lambda^{0}_{\sigma} - u^{0}_{\sigma} - u^{0\nu}_{\sigma}\partial_{\nu} + \partial_{\nu} u^{0\nu}_{\sigma} + \pi^{(r)}_{\alpha}(S^{\alpha}_{(r)\sigma} - \psi^{\alpha}_{(r),\mu} a^{\mu}_{\sigma})\right]\zeta^{\sigma}_{\rho}\right\}$$
$$= \text{const} \qquad (\rho = 1, 2, \ldots, s) \tag{22}$$

In the case of a singular first-order Lagrangian, we can proceed in the same way to obtain similar results (the subscript being suppressed).

In non-Abelian gauge theory, the Lagrangian without ghosts violates unitarity and hence the effective Lagrangian is given by

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + B^a \partial_\mu A^{\mu a} + \frac{\alpha_0}{2} (B^a)^2 - \partial_\mu \bar{C}^a D^{\mu a}_b C^b \tag{23}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a_{bc} A^b_\mu A^c_\nu, \qquad D^a_{\mu b} = \delta^a_b \partial_\mu + f^a_{cb} A^c_\mu \tag{24}$$

where A^a_μ are Yang–Mills fields, C^a and $\bar{C}^a$ are odd ghost fields, and B^a are additional even fields. The BRST invariance of the effective action implies the conserved BRST charge (Gitman and Tyutin, 1990)

$$Q = \int d^3x \left(-F^{0\nu a} D^a_{\nu b} C^b + B^a D^{0a}_b C^b - \frac{1}{2} \partial^0 \bar{C}^a f^a_{be} C^b C^e \right) \tag{25}$$

The canonical momenta conjugate to the fields A^a_μ, B^a, C^a, and $\bar{C}^a$ are

$$\begin{gathered} \pi_{0a} = B^a, \qquad \pi_{ia} = F^a_{i0}, \qquad \pi_{B^a} = 0, \\ \pi_a = -\dot{\bar{C}}^a, \qquad \bar{\pi}_a = D^{0a}_b C^b \end{gathered} \tag{26}$$

respectively. The canonical Hamiltonian density is given by

$$\begin{aligned} \mathscr{H}_c = {} & \frac{1}{2} \pi^2_{ia} + A^{0a} D^a_{ib} \pi_{ib} + \frac{1}{4} F^{a2}_{ik} + \pi_a \bar{\pi}_a - \pi_a f^a_{be} A^{0b} C^e \\ & + \bar{C}^a \partial_i D^a_{ib} C^b - B^a \partial_i A^{ia} - \frac{\alpha_0}{2} (B^a)^2 \end{aligned} \tag{27}$$

There are two PC:

$$\phi^0_{1a} = \pi_{0a} - B^a \approx 0, \qquad \phi^0_{2a} = \pi_{B^a} \approx 0 \tag{28}$$

The total Hamiltonian is given by

$$H_T = H_c + H' = \int d^3x \left(\mathscr{H}_c + \lambda^a_1 \phi^0_{1a} + \lambda^a_2 \phi^0_{2a} \right) \tag{29}$$

The constraints (28) are second class, and so secondary constraints are absent.

Let us now consider only the transformation of Yang–Mills fields, fixing the ghost fields and additional fields in the BRST transformation,

$$\begin{gathered} \delta A^a_\mu = D^a_{\mu b} \theta^b, \qquad \delta \pi_{\mu a} = f^a_{bc} \pi_{\mu c} \theta^b \\ \delta C^a = \delta \bar{C}^a = \delta B^a = 0, \qquad \delta \pi_a = \delta \bar{\pi}_a = \delta \pi_{B^a} = 0 \end{gathered} \tag{30}$$

where $\theta^a = C^a\tau$ and τ is a Grassmann parameter. Under the transformation (30), the effective Lagrangian $\mathscr{L}_{\text{eff}}$ is variant

$$\delta\mathscr{L}_{\text{eff}} = F(\theta) + u^\mu_a\partial_\mu\theta^a + u_a\partial^2\theta^a = F(\theta) \\ + f^a_{bc}(\partial^\mu\bar{C}^a \cdot C^c - B^a A^{\mu c})\partial_\mu\theta^b + B^a\partial^2\theta^a \tag{31}$$

where $F(\theta)$ does not contain the derivatives of the θ^a. In this case, similar to deducing expression (22), we have

$$J^w = \int_V d^3x\left\{\left[\delta^a_b\frac{\delta H'}{\delta A^a_0} - u^0_b - u^{0\nu}_b\partial_\nu + \partial_\nu u^{0\nu} + \pi^\mu_a D^a_{\mu b}\right]C^b\right\} = \text{const} \tag{32}$$

Thus, we obtain the conserved PBRST charge (P stands for "partial")

$$Q^{(p)} = \int_V d^3x\,[\pi_{\mu a}D^{\mu a}_b C^b + f^a_{be}(\pi_a C^b C^e - B_a A^{0b}C^e) + \dot{B}^a C_a - B^a\dot{C}_a] \tag{33}$$

This conserved PBRST charge differs significantly from the conserved BRST charge (25).

Similarly, if we fix the gauge field A^a_μ and change only the ghost fields, the weak conservation law implies a trivial identity.

The effective Lagrangian (23) is invariant under the gauge-translation transformation, which implies the gauge-invariant conserved energy-momentum (Dai, 1987). But if we consider only the transformation of Yang–Mills fields, we can obtain other weak conserved quantities.

We have shown that for certain cases the GNI (or strong conservation laws) in phase space may be converted into the weak conserved charge along the trajectory of motion even if the Lagrangian is not invariant under the specific transformation. This algorithm differs from the usual first Noether theorem, where the invariance under a finite continuous group implies the conservation laws.

As is well known, BRST charge annihilates the vacuum; the conserved PBRST charge may also impose some supplementary conditions on physical states as well as BRST charge and ghost charge (Fukuda *et al.*, 1981, 1983).

For Yang–Mills theories with higher derivatives whose Lagrangian is given by (Saito *et al.*, 1989; Gitman and Tyutin, 1990)

$$\mathscr{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - cD_\sigma F^a_{\mu\nu}D^\sigma F^{\mu\nu a} \tag{34}$$

using the Faddeev–Popov trick to formulate its path integral quantization, one can obtain the effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_g - \frac{1}{2\alpha_0}(\partial_\mu A^{\mu a})^2 - \partial_\mu\bar{C}^a D^{\mu a}_b C^b \tag{35}$$

which can be also derived by using the Dirac theory of constrained systems (Gitman and Tyutin, 1990). The effective Lagrangian (35) is invariant under the BRS transformation, which implies the BRS charge. But if we consider only the gauge transformation of Yang–Mills fields, we can obtain another PBRS charge.

ACKNOWLEDGMENTS

The author would like to thank Prof. L. Zhang for his kind help. This project was supported by the National Natural Science Foundation of China under grant 19275009 and by the Beijing Natural Science Foundation of China under grant 91-A-010.

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